

Spontaneous Left-Right Symmetry Breaking in Supersymmetric Models with only Higgs Doublets

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Abstract

We studied the question of parity breaking in a supersymmetric left-right model, in which the left-right symmetry is broken with Higgs doublets (carrying $B - L = \pm 1$). Unlike the left-right symmetric models with triplet Higgs scalars (carrying $B - L = \pm 2$), in this model it is possible to break parity spontaneously by adding a parity odd singlet. We then discussed how neutrino mass of type III seesaw can be invoked in this model by adding extra fermion singlets. We considered simple forms of the mass matrices that are consistent with the unification scheme and demonstrate how they can reproduce the required neutrino mixing matrix. In this model, the baryon asymmetry of the universe is generated via leptogenesis. The required mass scales in the model is then found to be consistent with the gauge coupling unification.

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I. INTRODUCTION

The existence of massive neutrinos, the unknown origin of parity violation in the Standard Model (SM) and the hierarchy problem are some of the important motivations for physics beyond the SM. The most natural extension of the standard model that addresses these issues is the supersymmetric version of the left-right symmetric extension of the standard model, which will treat the left-handed and right-handed particles on equal footing, and the parity violation we observe at low energies would be due to the spontaneous breaking of the left-right symmetry at some high scale [1–9]. Another interesting feature of the left-right symmetric model is that the difference between the baryon number (B) and the lepton number (L) becomes a gauge symmetry, which leads to several interesting consequences.

In spite of the several virtues of the minimal supersymmetric left-right symmetric models (MSLRM), we are yet to arrive at a fully consistent model, from which we can descend down to the MSSM. One of the most important problems is the spontaneous breaking of left-right symmetry [10, 11]. There has been suggestions to solve this problem by introducing additional fields or higher dimensional operators or by going through a different symmetry breaking chain or breaking the left-right symmetry around the supersymmetry breaking scale [8, 10–15]. In some cases, this problem is cured through the introduction of a parity-odd singlet, but the soft susy breaking terms then lead to breaking of electromagnetic charge invariance. One interesting SUSYLR model is the minimal SUSYLR model, which has been studied extensively [10, 11, 16], and it has been found that global minimum of the Higgs potential is either charge violating or R -parity violating.

Recently we proposed yet another solution to the problem, which resembles the non-supersymmetric solution, relating the vacuum expectation values (vev's) of the left-handed and right-handed triplet Higgs scalars to the Higgs bi-doublet vev through a seesaw relation. We achieved this by introducing a bi-triplet and singlet Higgs scalars, and the vacuum that preserves both electric charge and R -parity can naturally be the global minimum of the full potential. In this article we are applying this idea of spontaneous left-right symmetry breaking at high scale in supersymmetric models with only doublet Higgs scalars. We extend the model with one singlet Higgs scalar, which breaks the left-right parity of the gauge groups at a high scale. The most attractive feature of the present model is that it does not allow any left-right symmetric solution to be a minimum of the potential. We also discuss the

question of neutrino masses via type III see-saw mechanism and leptogenesis in details. We then embed the model in a grand unified theory and study the gauge coupling unification to check the consistency of the mass scales required in this model.

II. MINIMAL SUSYLR MODEL: A BRIEF REVIEW

In this section, we shall review the left-right extension of the standard model, where the gauge group at higher energies is the left-right symmetric group $G_{LR} \equiv SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and we assume that at energies above the TeV scale, the theory is supersymmetric. In these supersymmetric left-right symmetric models, it is assumed that the MSSM gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ is enhanced at some higher energy, above which the left-handed and right-handed fermions are treated on equal footing. The minimal supersymmetric left-right (SUSYLR) model starts with the left-right symmetric gauge group G_{LR} , which could emerge from a supersymmetric $SO(10)$ grand unified theory. The field content of this model is given by,

$$\begin{aligned} Q &= (3, 2, 1, 1/3), & Q^c &= (3, 1, 2, -1/3), \\ L &= (1, 2, 1, -1), & L^c &= (1, 1, 2, 1), \end{aligned} \quad (1)$$

where the numbers in the brackets denote the quantum numbers under G_{LR} .

The Higgs sector of this model consists of the bidoublet and triplet superfields, given by,

$$\begin{aligned} \Phi_i &= (1, 2, 2, 0), & (i = 1, 2), \\ \Delta &= (1, 3, 1, 2), & \bar{\Delta} = (1, 3, 1, -2), \\ \Delta^c &= (1, 1, 3, -2), & \bar{\Delta}^c = (1, 1, 3, 2). \end{aligned} \quad (2)$$

Under the left-right parity corresponding to the interchange of the gauge groups $SU(2)_L$ and $SU(2)_R$, or the D-parity, the fields transform as

$$\begin{aligned} Q &\leftrightarrow Q^{*c}, & L &\leftrightarrow L^{*c}, \\ \Delta &\leftrightarrow \Delta^{*c}, & \bar{\Delta} &\leftrightarrow \bar{\Delta}^{*c}, \\ \Phi_i &\leftrightarrow \Phi_i^\dagger. \end{aligned} \quad (3)$$

The superpotential for this theory is given by

$$\begin{aligned}
W = & Y^{(i)q} Q^T \tau_2 \Phi_i \tau_2 Q^c + Y^{(i)l} L^T \tau_2 \Phi_i \tau_2 L^c \\
& + i(f L^T \tau_2 \Delta L + f^* L^{cT} \tau_2 \Delta^c L^c) \\
& + \mu_\Delta \text{Tr}(\Delta \bar{\Delta}) + \mu_\Delta^* \text{Tr}(\Delta^c \bar{\Delta}^c) + \mu_{ij} \text{Tr}(\tau_2 \Phi_i^T \tau_2 \Phi_j).
\end{aligned} \tag{4}$$

One of the important problems with the supersymmetric left-right extension of the standard model is that the minimization of the potential does not allow spontaneous parity breaking, which was considered to be one of the major triumph of the non-supersymmetric LR models. Several attempts were made to solve this problem in some variants of the model. Some of these solutions involve modifying the Higgs sector, adding higher dimensional operators or involving a different breaking scheme of the group theory [12–17]. The simplest solution is to include a bi-triplet field [17] and allow D-parity breaking at some high scale, which may then allow parity violation spontaneously, allowing the scale of $SU(2)_R$ breaking to be different from the $SU(2)_L$ breaking scale. We extend that argument to the models involving only doublets.

In models with only doublet scalars, we require three singlet fermions to give masses to the neutrinos. The charged fermion masses originate from the vev of the bi-doublet scalar field. Since there are no triplet scalar field that breaks the symmetry $SU(2)_R$, and all the triplet scalars are replaced by the doublet scalars, the bi-doublet field required to give masses to the charged fermions can give rise to the coupling required to break the parity, when D-parity is broken. Thus left-right symmetry breaking becomes more natural in these models with only doublet scalar fields. This model is also able to generate baryon asymmetry via leptogenesis and provide neutrino masses through both inverse see-saw mechanism and also using Type-III see-saw mechanism.

III. SUSYLR WITH HIGGS DOUBLETS AND PARITY ODD SINGLET

We consider here a SUSYLR model with only doublet Higgs scalars, which is the simplest extension of the non-supersymmetric LR model. This includes the bi-doublet scalar field that is required to give masses to the charged fermions and also to break the $SU(2)_L$ symmetry after the left-right symmetry is broken. The doubling of the bidoublet Higgs in previous models was to ensure a non-vanishing CKM matrix. For the sake of simplicity of our model

we forgo this condition since it doesn't have any bearing on parity breaking. However, extension of the present model via doubling of the bidoublet is fairly trivial. Thus, the Higgs sector of our model is given by,

$$\begin{aligned}\chi_L &\equiv (1, 2, 1, -1), & \bar{\chi}_L &\equiv (1, 2, 1, 1), \\ \chi_R &\equiv (1, 1, 2, -1), & \bar{\chi}_R &\equiv (1, 1, 2, 1), \\ \Phi &= (1, 2, 2, 0), & \sigma &\equiv (1, 1, 1, 0).\end{aligned}\tag{5}$$

where, with usual custom the subscript L and R denotes the left and right handedness of the Higgs particle. The Higgs particles with “bar” in the notation, helps in anomaly cancellation of the model.

We have also included a singlet scalar field σ , which has the special property that it is even under the usual parity of the Lorentz group, but it is odd under the parity that relates the gauge groups $SU(2)_L$ and $SU(2)_R$. This field σ is thus a scalar and not a pseudo-scalar field, but under the D-parity transformation that interchanges $SU(2)_L$ with $SU(2)_R$, it is odd. This kind of work is proposed in [18, 19]. Although all the scalar fields are even under the parity of the Lorentz group, under the D-parity the Higgs sector transforms as,

$$\begin{aligned}\chi_L &\leftrightarrow \chi_R, & \bar{\chi}_L &\leftrightarrow \bar{\chi}_R, \\ \Phi &\leftrightarrow \Phi^\dagger, & \sigma &\leftrightarrow -\sigma.\end{aligned}\tag{6}$$

The superpotential of the model relevant in the context of parity breaking is given by,

$$\begin{aligned}W &= f\Phi(\bar{\chi}_L\chi_R + \chi_L\bar{\chi}_R) + m_\Phi\Phi\Phi \\ &\quad + m_\chi(\bar{\chi}_L\chi_L + \bar{\chi}_R\chi_R) \\ &\quad + m_\sigma\sigma^2 + \lambda\sigma(\bar{\chi}_L\chi_L - \bar{\chi}_R\chi_R).\end{aligned}\tag{7}$$

Supersymmetry being unbroken, implies the F and D conditions are equal to zero. The F flatness conditions for the various Higgs fields are given by,

$$\begin{aligned}F_\Phi &= f(\bar{\chi}_L\chi_R + \chi_L\bar{\chi}_R) + 2m_\Phi\Phi = 0, \\ F_{\chi_L} &= f\Phi\bar{\chi}_R + m_\chi\bar{\chi}_L + \lambda\sigma\bar{\chi}_L = 0, \\ F_{\bar{\chi}_L} &= f\Phi\chi_R + m_\chi\chi_L + \lambda\sigma\chi_L = 0, \\ F_{\chi_R} &= f\Phi\bar{\chi}_L + m_\chi\bar{\chi}_R - \lambda\sigma\bar{\chi}_R = 0, \\ F_{\bar{\chi}_R} &= f\Phi\chi_L + m_\chi\chi_R - \lambda\sigma\chi_L = 0, \\ F_\sigma &= 2m_\sigma\sigma + \lambda(\bar{\chi}_L\chi_L - \bar{\chi}_R\chi_R).\end{aligned}\tag{8}$$

Similarly, the D flatness conditions, are given by,

$$\begin{aligned}
D_{R_i} &= \chi_R^\dagger \tau_i \chi_R + \bar{\chi}_R^\dagger \tau_i \bar{\chi}_R = 0, \\
D_{L_i} &= \chi_L^\dagger \tau_i \chi_L + \bar{\chi}_L^\dagger \tau_i \bar{\chi}_L = 0, \\
D_{B-L} &= (\chi_L^\dagger \chi_L - \bar{\chi}_L^\dagger \bar{\chi}_L) - (\chi_R^\dagger \chi_R - \bar{\chi}_R^\dagger \bar{\chi}_R) = 0.
\end{aligned} \tag{9}$$

In both the F and D flat conditions we have neglected the lepton fields, since they would have a zero vev. The vev's for the scalar fields are given by,

$$\begin{aligned}
\langle \chi_L \rangle &= \langle \bar{\chi}_L \rangle = v_L, \\
\langle \chi_R \rangle &= \langle \bar{\chi}_R \rangle = v_R, \\
\langle \Phi \rangle &= v, \quad \langle \sigma \rangle = s.
\end{aligned} \tag{10}$$

Here, for simplicity of the model, we have assumed χ_L and $\bar{\chi}_L$ to have the same vev v_L . Similarly, for the right-handed fields χ_R and $\bar{\chi}_R$.

Minimization of D flat conditions, leads to a number of holomorphic gauge invariants which corresponds to flat directions [20]. Here, however, in order to determine the vacuum structure of our model, we minimize the F flat conditions and discuss about the relations that emerge from them.

After the scalar fields have acquired their respective vevs, the F flatness conditions are given by,

$$F_\Phi = f(v_L v_R + v_R v_L) + 2m_\Phi v = 0, \tag{11}$$

$$F_{\chi_L} = f v v_R + \lambda s v_L + m_\chi v_L = 0, \tag{12}$$

$$F_{\bar{\chi}_L} = f v v_R + \lambda s v_L + m_\chi v_L = 0, \tag{13}$$

$$F_{\chi_R} = f v v_L - \lambda s v_R + m_\chi v_R = 0, \tag{14}$$

$$F_{\bar{\chi}_R} = f v v_L - \lambda s v_R + m_\chi v_R = 0, \tag{15}$$

$$F_\sigma = 2m_\sigma s + \lambda(v_L^2 - v_R^2) = 0. \tag{16}$$

Solving the equations we get four relations among the vevs.

$$v_L = \frac{-m_\Phi v}{f v_R} \quad (17)$$

$$m_\chi + \lambda s = \frac{f v v_R}{v_L} \quad (18)$$

$$m_\chi - \lambda s = -\frac{f v v_L}{v_R} \quad (19)$$

$$s = \frac{\lambda}{2m_\sigma}(v_R^2 - v_L^2) \quad (20)$$

The role of D-parity odd singlets σ is uni-important in left-right breaking. This can be understood from eqns. (18) and (19) as follows:

$$\left(\frac{v_L}{v_R}\right)^2 = \frac{M - \lambda s}{M + \lambda s} \quad (21)$$

If there is no σ field, then $s = 0$. This implies $v_L = v_R$ which is a left-right symmetric solution. Also the F-term conditions (12)-(15) are not consistent without the inclusion of the parity odd singlet σ in the model. Hence, the parity odd singlet σ is necessary to account for the spontaneous left-right breaking and for the consistency of the model.

We now try to interpret these results to get a working phenomenology. Considering the last of the relations eqn (20) we see that $s = 0$ is a trivial solution, and will put v_L and v_R on equal footing thus leading to unbroken parity. However, $s = 0$ is a special solution of eqn (20). For $s \neq 0$, we have $v_L \neq v_R$ and parity is violated spontaneously. We will choose $v_R \gg v_L$, as it is usually assumed in model building for phenomenological reasons. Choosing the mass (m_Φ) and vev (v) of Φ to be of electroweak (EW) scale and considering the dimensionless coupling constant λ to be of order unity, we immediately come to the conclusion, from eqn (19), that $m_\chi \sim s$.

In order to avoid generic susy problems like over abundance of gravitino, we assume the mass scale of v_R to be $\leq 10^9$ GeV. This together with eqn (17) gives the value of $v_L \simeq 10^{-5}$ GeV, where f , another dimensionless quantity, without any fine-tuning is considered to be of order unity. This is also consistent with the assumption that $v_R \gg v_L$. Now using eqn (18) and the above derived relation that $m_\chi \sim s$ we get $m_\chi \sim s \simeq 10^{16}$ GeV. Finally, from eqn (20) one derives the mass of σ (m_σ) to be of EW scale. If one considers non-thermal leptogenesis, then one can consider the alternative possibility of having a low value of v_R i.e. $\sim \mathcal{O}(10)$ TeV. Then all the mass scales and vevs are reduced by a couple of orders and could be accessible to colliders. The results are summarized in Table (I).

Masses/VeVs	Case - I (In GeV)
m_χ, s	10^{16}
v_R	10^9
m_Φ, v, m_σ	10^2
v_L	10^{-5}

TABLE I: Mass scales of the model

IV. NEUTRINO MASS AND LEPTOGENESIS IN SUSYLR MODEL WITH HIGGS DOUBLET

In LR models with only doublet scalar fields, the question of neutrino masses and leptogenesis has been discussed in details. We shall try to restrict ourselves as close as possible to these existing non-supersymmetric models, and check the consistency of these solutions when parity is broken in the present SUSYLR model. We shall first discuss the scenario with conserved D-parity, but since LR symmetry cannot be broken without breaking D-parity we shall discuss the D-parity breaking scenario afterwards.

In conventional type I seesaw, neutrino mass can be realized via three right handed neutrinos N_i^c where we have Majorana mass term $(M_R)_{ij} N_i^c N_j^c$ and Dirac masses with the ordinary neutrinos $(M_N)_{ij} \nu_i N_j^c = (Y_N)_{ij} \nu_i N_j^c \langle \Phi \rangle$. After diagonalizing, the resulting neutrino mass is $M_\nu^I = -M_N M_R^{-1} M_N^T$. Type II seesaw requires a $SU(2)_L$ triplet Higgs field T with mass of order m_T . Integrating out the Higgs triplet T leads to a mass operator $(M_T)_{ij} \nu_i \nu_j$ with $M_T \propto \frac{Y_T \langle \Phi \rangle^2}{m_T} \sim \frac{v^2}{M_G}$. Combination of these neutrino mass are also possible in left-right models which contains both type I and type-II or, type I and type III [21, 22].

In type III neutrino mass [23] three hypercharge neutral fermionic triplets Σ^a ($a = 1, 2, 3$) are added to explain the ν mass term. In our model, however, we have an extra fermionic superfield which give rise ν mass term which is similar to the conventional type III seesaw mechanism. Thus, it is in this spirit that we can call the seesaw mechanism in our model as type III seesaw. For the review of the standard type III seesaw mechanism we closely follow [24].

Along with the Dirac neutrino mass term $(M_N)_{ij} \nu_i N_j^c$, the relevant superpotential for ν

mass term, which is due to the extra fermion singlet (S) is given by,

$$W = M_{ij}S_iS_j + F_{ij}l_{Li}S_j\chi_L + F'_{ij}l_{Ri}S_j\chi_R, \quad (22)$$

From the above superpotential one can see that the vev of the left-handed doublet Higgs field which acquires a low scale vev $\langle\chi_L\rangle = v_L$ directly couples the left-handed ν'_i s with the singlet S_i . The mass matrix for the neutral leptons has the form,

$$W_{\text{neut}} = (\nu_i \quad N_i^c \quad S_i) \begin{pmatrix} 0 & (M_N)_{ij} & F_{ij}v_L \\ (M_N)_{ji} & 0 & F_{ij}v_R \\ F_{ji}v_L & F_{ji}v_R & \mathcal{M}_{ij} \end{pmatrix} \begin{pmatrix} \nu_j \\ N_j^c \\ S_j \end{pmatrix}. \quad (23)$$

In the above mass matrix, the mass of the singlet \mathcal{M}_{ij} and the vev of the right-handed Higgs doublet v_R are heavy, while M_N and vev of the left-handed Higgs doublet v_L are of low scale.

Since in our model we have more than one left-handed Higgs doublet ($\chi_L, \bar{\chi}_R$), the ν mass is given by,

$$M_\nu = -M_N M_R^{-1} M_N^T - (M_N H + H^T M_N^T) \left(\frac{v_L}{v_R} \right), \quad (24)$$

$$\text{where,} \quad H \equiv (F' \cdot F^{-1})^T, \quad (25)$$

$$M_R = (F v_R) \mathcal{M}^{-1} (F^T v_R). \quad (26)$$

The first term in eqn (24) is the type I seesaw contribution and the second term gives the type III seesaw contribution. Type III contribution to ν mass will dominate over type I if the elements of the matrix \mathcal{M}_{ij} are small compared to the contribution of H term.

We will partly follow the formalism and parametrization used in [24, 25] where the elements of the Dirac mass matrix are $M_{N11} = \eta v$, $M_{N33} = v$, $M_{N23} = -M_{N32} = v\epsilon$ and else are zero. Here $\eta = 0.6 \times 10^{-5}$ and $\epsilon \sim 0.14$.

If the elements of F_{ij} and F'_{ij} are considered to be of the order of f , a dimensionless parameter then from eqn. (25) we find that $H_{ij} \sim 1$ ($i, j = 1, 2, 3$). Thus, the ν mass resulting from eqn (24) is

$$M_\nu = \begin{pmatrix} \eta & \epsilon & 1 \\ \epsilon & \epsilon & 1 \\ 1 & 1 & 1 \end{pmatrix} \frac{v v_L}{v_R} \quad (27)$$

The neutrino mass as presented above mostly satisfy the observed neutrino mass with a minor fine tuning in the 13 element.

Another set of parameters can be chosen to explain both neutrino mass and leptogenesis where both F_{ij} and F'_{ij} take the form [24]

$$F, F' \sim \begin{pmatrix} \lambda^2 & \lambda & \lambda \\ \lambda & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix}, \quad (28)$$

where $\lambda \sim \eta/\epsilon$. With this form of F, F' we have from eqns (24) and (28),

$$H \sim \begin{pmatrix} 1 & \epsilon/\eta & \epsilon/\eta \\ \eta/\epsilon & 1 & 1 \\ \eta/\epsilon & 1 & 1 \end{pmatrix}, \quad (29)$$

and

$$M_\nu \sim \begin{pmatrix} \eta & \epsilon & \epsilon \\ \epsilon & \epsilon & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \frac{v v_L}{v_R}. \quad (30)$$

For the study of leptogenesis, a diagonal F_{ij} would suffice better. The parameters in this new basis would be represented via a tilde. The right-handed neutrino and the singlet has to be transformed via a unitary transformation to attain the diagonal basis as such $N_i^c = U_{ij} \tilde{N}_j^c$ and $S_i = V_{ij} \tilde{S}_j$. To attain the diagonal form of F_{ij} the unitary matrix U_{ij} can have the form

$$U = \begin{pmatrix} u_{11} & \lambda u_{12} & \lambda u_{13} \\ \lambda u_{21} & u_{22} & u_{23} \\ \lambda u_{31} & u_{32} & u_{33} \end{pmatrix} \quad (31)$$

with V_{ij} having a similar form. Here the u_{ij} elements are of $\mathcal{O}(1)$. For simplicity and numerical computation we will use the particular form of the unitary matrix which is

$$U = \begin{pmatrix} 1 & -\lambda(1 + \sqrt{2})i & \lambda \\ -\lambda(1 + \sqrt{2})i & 1/\sqrt{2} & i/\sqrt{2} \\ \lambda & i/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}. \quad (32)$$

The elements of the diagonalized matrix $\tilde{F}_{ij} v_R = (U_{ki} F_{k\ell} V_{\ell j}) v_R$ can be written

$$\tilde{F} v_R = \text{diag}[\lambda^2 F_1, F_2, F_3] v_R \equiv \text{diag}[M_1, M_2, M_3], \quad (33)$$

where $F_i \sim 1$. In this basis the matrices $\tilde{F}'_{ij}u$ and $\tilde{\mathcal{M}}_{ij}$ can be parametrized as

$$\begin{aligned}\tilde{F}'u &= \begin{pmatrix} \lambda^2 f_{11} & \lambda f_{12} & \lambda f_{13} \\ \lambda f_{21} & f_{22} & f_{23} \\ \lambda f_{31} & f_{32} & f_{33} \end{pmatrix} v, \\ \tilde{\mathcal{M}} &= \begin{pmatrix} \lambda^2 g_{11} & \lambda g_{12} & \lambda g_{13} \\ \lambda g_{21} & g_{22} & g_{23} \\ \lambda g_{31} & g_{32} & g_{33} \end{pmatrix} M_S,\end{aligned}\tag{34}$$

where, $f_{ij}, g_{ij} \sim 1$. The assumption here is that the scale of $M_S \ll v_R$. In the new basis, the Dirac neutrino mass matrix M_N transforms as $\tilde{M}_N = M_N U$ and the form of the transformed matrix is

$$\tilde{M}_N \cong \begin{pmatrix} \eta u_{11} & \eta \lambda u_{12} & \eta \lambda u_{13} \\ \epsilon \lambda u_{31} & \epsilon u_{32} & \epsilon u_{33} \\ \lambda u_{31} & u_{32} & u_{33} \end{pmatrix} v \equiv \tilde{Y} v.\tag{35}$$

After doing all the parametrization, the type III seesaw contribution to the light neutrino mass matrix (which dominates, since $M_S \ll v_R$) from eqn (24) is given by,

$$M_\nu \cong - \begin{bmatrix} 2\eta \left(\frac{u_{11}f_{11}}{F_1} \right) & \frac{\eta}{\lambda} \left(\frac{u_{11}f_{21}}{F_1} \right) & \frac{\eta}{\lambda} \left(\frac{u_{11}f_{31}}{F_1} \right) \\ \frac{\eta}{\lambda} \left(\frac{u_{11}f_{21}}{F_1} \right) & 2\epsilon \sum_j \left(\frac{u_{3j}f_{2j}}{F_j} \right) & \sum_j \left(\frac{u_{3j}f_{2j}}{F_j} \right) \\ \frac{\eta}{\lambda} \left(\frac{u_{11}f_{31}}{F_1} \right) & \sum_j \left(\frac{u_{3j}f_{2j}}{F_j} \right) & 2\sum_j \left(\frac{u_{3j}f_{3j}}{F_j} \right) \end{bmatrix} \left(\frac{v^2}{v_R} \right).\tag{36}$$

Now we discuss the leptogenesis scenario in the given form of the neutrino matrix M_N , \mathcal{M} , M_S and U [24, 25]. Consider the case where the six super heavy two-component neutrinos have the mass matrix

$$(\tilde{N}_i^c, \tilde{S}_i) \begin{pmatrix} 0 & M_i \delta_{ij} \\ M_i \delta_{ij} & \tilde{\mathcal{M}}_{ij} \end{pmatrix} \begin{pmatrix} \tilde{N}_j^c \\ \tilde{S}_j \end{pmatrix},\tag{37}$$

where, $\tilde{\mathcal{M}}_{ij}$ is given in eqn (34). The leptogenesis can be realized by the decays of the lightest pair of these super heavy neutrinos, which have effectively the 2×2 mass matrix

$$(\tilde{N}_1^c, \tilde{S}_1) \begin{pmatrix} 0 & M_1 \\ M_1 & \tilde{\mathcal{M}}_{11} \end{pmatrix} \begin{pmatrix} \tilde{N}_1^c \\ \tilde{S}_1 \end{pmatrix} = (\tilde{N}_1^c, \tilde{S}_1) \lambda^2 \begin{pmatrix} 0 & F_1 v_R \\ F_1 v_R & g_{11} M_S \end{pmatrix} \begin{pmatrix} \tilde{N}_1^c \\ \tilde{S}_1 \end{pmatrix}.\tag{38}$$

Consider the scenario where $M_S \ll v_R$, then this results an almost degenerate pseudo-Dirac pair or equivalently two Majorana neutrinos with nearly equal and opposite masses. These

Majorana neutrinos are $N_{\pm} \cong (\tilde{N}_1^c \pm \tilde{S}_1)/\sqrt{2}$, with masses $M_{\pm} \cong \pm M_1 + \frac{1}{2}\tilde{\mathcal{M}}_{11} = \lambda^2(\pm F_1 v_R + \frac{1}{2}g_{11}M_S)$. These can decay into light neutrino plus Higgs via the term $Y_{i\pm}(N_{\pm}\nu_i)H$, where

$$Y_{i\pm} \cong (\tilde{Y}_{i1} \pm \tilde{F}'_{i1})/\sqrt{2} \mp \frac{\tilde{\mathcal{M}}_{11}}{4M_1}(\tilde{Y}_{i1} \mp \tilde{F}'_{i1})/\sqrt{2}. \quad (39)$$

Here \tilde{Y} is the Dirac Yukawa coupling matrix given in eqn (35). It is straightforward to show that the lepton asymmetry produced by the decays of N_{\pm} [24] is given by

$$\epsilon_1 = \frac{1}{4\pi} \frac{\text{Im}[\sum_j (Y_{j+} Y_{j-}^*)]^2}{\sum_j [|Y_{j+}|^2 + |Y_{j-}|^2]} I(M_-^2/M_+^2), \quad (40)$$

where $f(M_{1+}^2/M_{1-}^2)$ comes from the absorptive part of the decay amplitude of N_{\pm} . This function is given by

$$I(x) = \sqrt{x} \left[\frac{1}{1-x} + 1 - (1+x) \ln \left(\frac{1+x}{x} \right) \right] \quad (41)$$

Making use of eqns (39) and (40) one obtains

$$\begin{aligned} \epsilon_1 &= \frac{1}{4\pi} \frac{\sum_j (|\tilde{Y}_{j1}|^2 - |\tilde{F}'_{j1}|^2) \text{Im}(\sum_k \tilde{Y}_{k1}^* \tilde{F}'_{k1})}{\sum_j (|\tilde{Y}_{j1}|^2 + |\tilde{F}'_{j1}|^2)} f(M_{1+}^2/M_{1-}^2), \\ \text{or, } \epsilon_1 &\cong \frac{\lambda^2}{4\pi} \left[\frac{(|u_{31}|^2 - |f'_{31}|^2) \text{Im}(u_{31}^* f'_{31})}{|u_{31}|^2 + |f'_{31}|^2 + |f'_{21}|^2} \right] f(M_{1+}^2/M_{1-}^2). \end{aligned} \quad (42)$$

The lepton asymmetry produced by the decay on lightest Majorana neutrino is partially diluted by the lepton number violating decay processes. This decay processes try to wash out the lepton asymmetry already produce before. This wash out factor is given by,

$$k(\tilde{m}_1) \sim 0.3 \left(\frac{10^{-3} \text{eV}}{\tilde{m}_1} \right) \left(\log \frac{\tilde{m}_1}{10^{-3} \text{eV}} \right)^{-0.6} \quad (43)$$

The equilibrium mass of the neutrino is given by

$$\tilde{m}_1 \equiv \frac{8\pi v_u^2 \Gamma_{N_{1\pm}}}{M_{N_{1\pm}}^2} \cong \lambda^2 \frac{v_u^2}{M_1} (|u_{31}|^2 + |f'_{31}|^2 + |f'_{21}|^2). \quad (44)$$

A. Numerical Result

The lepton asymmetry produced per unit entropy, taking into account decays of Majorana neutrino and their washout factors, is given by

$$\begin{aligned} \frac{n_L}{s} &\cong \frac{k}{s} \frac{\epsilon_1}{\pi^2} \frac{g_N T^3}{\pi^2} \\ &\cong \frac{45}{2\pi^4} \frac{g_N}{g_*} k \epsilon_1 \end{aligned} \quad (45)$$

Input	Case (III-1)	Case (III-2)	Case (III-3)	Case (III-4)
v_R (GeV)	2.7×10^{14}	2.7×10^{12}	8.8×10^{10}	9.8×10^8
F_1	1.0	10.	31	50
F_2	1.0	0.1	0.1	1.0
F_3	1.0	1.0	1.0	1.0
M_S (GeV)	4.3×10^5	430	43	10.0
f_{21}	$-0.950 + 0.534i$	$-0.050 + 0.0534 i$	$-0.950 + 0.11 i$	$-0.01+0.01 i$
f_{22}	$-2.279 - 1.537i$	$-0.227 - 0.154i$	$-0.228 - 0.154i$	$-0.225+0.138 i$
f_{23}	$-0.194 + 1.523i$	$-0.194 + 1.523i$	$-0.193 + 0.573 i$	$-0.195 + 1.23 i$
f_{31}	$0.6+3.5 i$	$-0.012 + 0.385 i$	$-0.46 + 0.42 i$	$0.04 +0.04 i$
f_{32}	$-0.354i$	$-0.035i$	$-0.035i$	$0.023 i$
f_{33}	0.354	0.354	0.354	0.523

TABLE II: Type III seesaw and Leptogenesis results for four cases

We have used the expression for entropy of the comoving volume, $s = \frac{2}{45} g_* \pi^2 T^3$. Here $g_N = 2$ for Majorana spin degrees freedom and $g_* = 228.75$ is the relativistically spin degrees of freedom for supersymmetry.

The corresponding B-L asymmetry per unit entropy is just the negative of n_L/s , since baryon number is conserved in the right-handed Majorana neutrino decays. While $B - L$ is conserved by the electroweak interaction following those decays, the sphaleron processes violate $B + L$ conservation and convert the $B - L$ asymmetry into a baryon asymmetry. The baryon asymmetry for supersymmetric case is

$$\frac{n_B}{s} = -\frac{28}{79} \frac{n_L}{s} \quad (46)$$

With the entropy density $s = 7.04 n_\gamma$ in terms of the photon density, the baryon asymmetry(η_B) of the Universe, defined by the ratio n_B of the net baryon number to the photon number, is given in terms of the lepton asymmetry(ϵ_1) and washout parameter (k)

Output	Case (III-1)	Case (III-2)	Case (III-3)	Case (III-4)
M_1 (GeV)	4.53×10^5	4.53×10^3	4.58×10^3	82.37
M_2 (GeV)	2.70×10^{14}	2.70×10^{12}	8.8×10^{10}	9.8×10^8
M_3 (GeV)	2.70×10^{14}	2.70×10^{12}	8.8×10^{10}	9.8×10^8
$(M_{1+} + M_{1-})/M_{1+}$	1.6×10^{-9}	1.59×10^{-10}	1.57×10^{-10}	4.08×10^{-9}
ϵ_1	-2.5×10^{-6}	-2.1×10^{-4}	-1.01×10^{-6}	-1.01×10^{-4}
\tilde{m}_1 (eV)	0.511	0.569	4.774	0.694
κ_1	5.1×10^{-4}	4.5×10^{-4}	4.5×10^{-5}	3.6×10^{-4}
η_B	1.11×10^{-10}	1.147×10^{-10}	3.911×10^{-10}	1.461×10^{-10}

TABLE III: Type III seesaw results for four cases

by

$$\eta_B = \frac{n_B}{n_\gamma} \cong -0.039 k \epsilon_1. \quad (47)$$

Successful Leptogenesis will require that the final result for η_B should be order of 10^{10} . where $\lambda = \eta/\epsilon = 4.1 \times 10^{-5}$ as before.

The input parameter given in the table (II) which will determine the small neutrino mass, leptogenesis parameter as output given in the table (III) of our model.

V. GAUGE COUPLING UNIFICATION

Grand unified theories (GUTs) offer the possibility of unifying the three gauge groups viz., $SU(3)$, $SU(2)$ and $U(1)$ of the standard model into one large group at a high energy scale M_U . This scale is determined as the intersection point of the $SU(3)$, $SU(2)$ and $U(1)$ couplings. The particle content of the theory completely determines the variation of the couplings with energy. Given the particle content of the theory one can evolve the couplings, determined at low energies, to determine whether there is unification or not.

In this section we will discuss how one can obtain $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} (g_L = g_R) (\cong G_{2213})$ intermediate gauge symmetry in R-parity conserving supersymmetric grand unified theory through one-loop unification of gauge couplings. Suppose

we want to evolve coupling parameter between the scales M_1 and M_2 (i.e, $M_1 \leq \mu \leq M_2$) corresponding to the two scales of physics, then the RGE's depend on the gauge symmetry and particle content at $\mu = M_1$. In table (IV), we give the particle content of the model.

Fields	$SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
Q	$(3, 2, 1, +1/3)$
Q^c	$(3^*, 1, 2, -1/3)$
L	$(1, 2, 1, -1)$
L^c	$(1, 1, 2, +1)$
χ_L	$(1, 2, 1, +1)$
χ_R	$(1, 1, 2, -1)$
$\bar{\chi}_L$	$(1, 2, 1, -1)$
$\bar{\chi}_R$	$(1, 1, 2, +1)$
Φ_a	$(1, 2, 2, 0)$
S	$(1, 1, 1, 0)$

TABLE IV: Field content of the SUSY LR model

For this purpose, we consider the two step breaking of the group G to the minimal supersymmetric standard model (MSSM) through G_{3221} intermediate gauge symmetry in the so called minimal grand unified theory.

$$\begin{aligned}
G &\xrightarrow{M_U} SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} [G_{3221}] \\
&\xrightarrow{M_R} SU(3)_c \times SU(2)_L \times U(1)_Y [G_{321}] \\
&\xrightarrow{M_W} SU(3)_c \times U(1)_Q [G_{em}].
\end{aligned}$$

A. RGE for SUSYLR model with doublet Higgs

The couplings evolve according to their respective beta functions. The renormalization group equations(RGEs) for this model can be written as

$$\frac{d\alpha_i}{dt} = \alpha_i^2 [b_i + \alpha_j b_{ij} + O(\alpha^2)] \quad (48)$$

where, $t = 2\pi \ln(\mu)$. The indices $i, j = 1, 2, 3$ refer to the gauge group $U(1)$, $SU(2)$ and $SU(3)$ respectively.

Unlike the D-parity breaking case where the intermediate left-right gauge group has four different coupling constants as discussed in [26], in the present case G_{3221} has only three gauge couplings, $g_{2L} = g_{2R}$, g_{3C} , and g_{BL} for $\mu \geq M_R$. We now write down the RG evolution equation of gauge couplings upto one loop order which are given below

$$\begin{aligned}\frac{1}{\alpha_Y(M_Z)} &= \frac{1}{\alpha_G} + \frac{a_Y}{2\pi} \ln \frac{M_R}{M_Z} + \frac{1}{10\pi} (3a'_{2L} + 2a'_{BL}) \ln \frac{M_U}{M_R}, \\ \frac{1}{\alpha_{2L}(M_Z)} &= \frac{1}{\alpha_G} + \frac{a_{2L}}{2\pi} \ln \frac{M_R}{M_Z} + \frac{a'_{2L}}{2\pi} \ln \frac{M_U}{M_R}, \\ \frac{1}{\alpha_{3C}(M_Z)} &= \frac{1}{\alpha_G} + \frac{a_{3C}}{2\pi} \ln \frac{M_R}{M_Z} + \frac{a'_{3C}}{2\pi} \ln \frac{M_U}{M_R}.\end{aligned}\tag{49}$$

where $\alpha_G = g_G^2/4\pi$ is the GUT fine-structure constant and the beta function coefficients a_i and a'_i are determined by the particle spectrum in the ranges from M_Z to M_R , and from M_R to M_U , respectively.

Here we are using PDG values, $\alpha(M_Z) = 127.9$, $\sin^2 \theta_W(M_Z) = 0.2312$, and $\alpha_{3C}(M_Z) = 0.1187$ [27]. Consider the case where $SU(2)_R \times U(1)_{B-L}$ breaks down to $U(1)_Y$. In that case

$$\frac{Y}{2} = I_{3,R} + \frac{B-L}{2}\tag{50}$$

The normalized generators are $I_Y = (\frac{3}{5})^{1/2} \frac{Y}{2}$ and $I_{B-L} = (\frac{3}{2})^{1/2} \frac{B-L}{2}$. Using these, one can write

$$I_Y = \sqrt{\frac{3}{5}} I_{3,R} + \sqrt{\frac{2}{5}} I_{B-L}\tag{51}$$

Which implies that the matching of the coupling constant at the scale where the left-right symmetry begins to manifest itself is given by

$$\alpha_Y^{-1} = \frac{3}{5} \alpha_{2R}^{-1} + \frac{2}{5} \alpha_{B-L}^{-1}\tag{52}$$

B. Result

1. At scale $\mu = M_Z - M_R$,

$$a_Y = 33/5, a_{2L} = 1, a_{3C} = -3,\tag{53}$$

2. At scale $\mu = M_R - M_U$,

$$\begin{aligned}b'_{BL} &= 16, b'_{2L} = b'_{2R} = 4, \\ a'_{3C} &= a_{3C} = -3.\end{aligned}\tag{54}$$

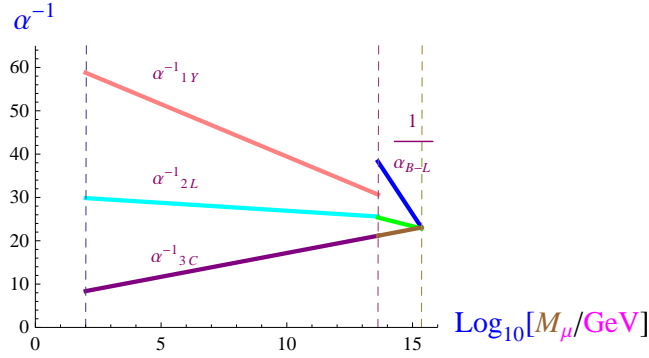


FIG. 1: Evolution of coupling constants in susylr model with Higgs doublet

This will change once we add contributions coming from extra particle added to the minimal supersymmetric model. Once we fix the values of beta functions, we can achieve lower values of M_R . There are discussion [28–30], where the Unification is possible at the same energy scale around 10^{16} GeV, but the scale of M_R varies from 10^9 - 10^{12} GeV.

Let us summarize our results. We point out that the non-supersymmetric version of the Standard Model is ruled out by LEP data. However, the supersymmetric extension of this scenario remains a viable alternative to conventional grand unified theories and is capable of predicting the precision values of couplings determined from LEP and unification is possible within the error bar. There are model [26, 31] where one can achieve unification of all three fundamental interactions in which D-parity is broken at the GUT level. We see from figure (1) that the gauge couplings unify at a scale 5.27×10^{15} GeV. Also the right handed scale M_R is found to be 2.69×10^{13} GeV in our model.

VI. CONCLUSION

We studied the question of spontaneous parity breaking in the supersymmetric version of the left-right symmetric models, in which all symmetry breaking takes place with only doublet Higgs scalars. We demonstrate that unlike the models with triplet Higgs scalars, in these models the left-right symmetry could be broken at a different scale compared to the electroweak symmetry breaking scale, if we introduce a singlet Higgs scalar σ , which breaks D-parity, that is the parity relating the gauge groups $SU(2)_L$ and $SU(2)_R$ but not relating to the parity of the Lorentz group. The vev of the field σ breaks the D-parity, but

does not break the Lorentz parity. But when combined with the vevs of the other doublet scalars, it allows to break the group $SU(2)_R$ at a different scale than the $SU(2)_L$ breaking scale, which is in the range of $10^8 - 10^{13}$ GeV, (though we can have low v_R which is allowed from minimization of the potential). We then demonstrated the consistency of the model in terms of the neutrino mass and the matter-antimatter asymmetry of the Universe. We then consider embedding of the model and check the consistency of the mass scales involved for the gauge coupling unification.

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